Analytic Mechanics: Discussion Worksheet 3b

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Consider a scalar function of two variables, $f(x, y)$. The partial derivative of a function f with respect to a variable x, $\frac{\partial f}{\partial x}$, is calculated by assuming that *all other variables* on which f depends are constants. This yields a new function which we'll call $f|_y(x)$ (read "f at constant y of x") which is dependent on x. We take a plain old derivative of $f|_y$ with respect to x, and that's $\frac{\partial f}{\partial x}$.

1 Given $f(x, y) = sin(x)sin(y)$, calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

2 Let's define $p = \frac{1}{\sqrt{2}}(x + y)$ and $q = \frac{1}{\sqrt{2}}(x - y)$.

2.1 On an $x-y$ plot, sketch the lines $p = 0$ and $q = 0$. Why do we assert that p and q are "independent coordinates?"

2.2 Write the f given previously in terms of p and q. That is, find a function $f_{new}(p, q)$ such that $f_{new}(p, q) = f(x, y)$ if p and q are defined as above.

2.3 Calculate $\frac{\partial f}{\partial p}$ and $\frac{\partial f}{\partial q}$ explicitly. How could you write expressions for these in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?

2.4 Suppose you have a coordinate s given by $s = \hat{n} \cdot \vec{r}$, $\vec{r} =< x, y >$. How would you calculate $\frac{\partial f}{\partial s}$, given $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?

3 What is the difference between the following expressions?

$$Q_1 = f(x_0, y_0) + \int_{x_0}^{x_f} \frac{\partial f}{\partial x} dx \quad (1)$$

$$Q_2 = f(x_0, y_0) + \int_{x_0}^{x_f} \frac{df}{dx} dx \quad (2)$$

3.1 Which of the two is "incomplete" – that is, requires the definition of another quantity to make it meaningful (assuming that f, x_0, and y_0 are all defined)?

3.2 Modify one of the expressions to make it identical to the other.

4 Define the vector $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \ldots\right)$, where for any number of dimensions, $\nabla_i \equiv \partial_i \equiv \frac{\partial}{\partial x_i}$. Show that $\vec{\nabla}$ is a vector (in the physics sense).

5 Invent at least four ways to combine the $\vec{\nabla}$ vector operator with one sort of field or another to construct scalar, vector, or tensor fields.

6 Suppose you have a vector field $\vec{F}(\vec{r})$. Now we perform a rotation $\hat{R}$ on the field and end up with a new field $\vec{F}'$. Give an expression for $\vec{F}'(\vec{r}')$.