Set #10

[7]

(a) $J_A \rightarrow$ elliptically pol (RH)

$J_B \rightarrow$ elliptically pol (LH)

(b)

$$J_A + J_B = \begin{pmatrix} \sqrt{2} \\ 1 \end{pmatrix}$$

Linearly pol.

(c) $$I = \sqrt{\frac{\epsilon}{\mu}} \frac{1}{2} \, \mathrm{Re} \left(\vec{E} \cdot \vec{E}_0^{\,*} \right)$$

$$I_{J_A} = I_{J_B} = \sqrt{\frac{\epsilon}{\mu}} \frac{1}{2}$$

$$I_{tot} = I_{J_A + J_B} = \sqrt{\frac{\epsilon}{\mu}} \frac{1}{2} \, 3 = 3 \, I$$

(d)

(d)

$$S_{J_A} = \begin{pmatrix} 1 \\ 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$S_{J_B} = \begin{pmatrix} 1 \\ 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$S_{J_A + J_B} = S_{J_A} + S_{J_B} = \begin{pmatrix} 2 \\ 0 \\ \sqrt{2} \\ 0 \end{pmatrix}$$

Linearly polarized.

(e) From PS 9 Problem 2(b),

$$\frac{I_p}{I_p + I_n} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

So fraction of natural light is $1 - \frac{1}{\sqrt{2}}$.

2

(a) Wave at Q at $t = t + \tau_{A,B}$ is same as advancing the wave of P at $t = t$ by $\int_P^Q \vec{k}_{A,B} \cdot d\vec{r}_{A,B}$. So

$$E_Q^{A,B}(t + \tau_{A,B}) = \vec{E}_P^{A,B}(t)\, e^{i \int_P^Q \vec{k}_{A,B} \cdot d\vec{r}_{A,B}}$$

$$= \vec{E}_P^{A,B}(t)\, e^{i \omega \tau_{A,B}}$$

(b) Just by simple substitution,

$$\vec{E}_Q^{B}(t - (\tau_B - \tau_A) + \tau_B) = \vec{E}_P^{B}(t - \tau_B + \tau_A)\, e^{i\omega\tau_B}$$

$$\downarrow$$

$$\vec{E}_Q^{B}(t + \tau_A) = \vec{E}_P^{B}(t - \tau)\, e^{i\omega\tau_B}$$

(c)

$$E_Q^{A*}(t+\tau_A) = E_p^{A*}(t)\, e^{-i\omega\tau_A}$$

So, from $2\sqrt{\frac{\mu}{\epsilon}}\, S = |E^A|^2 + |E^B|^2 + 2\,\mathrm{Re}(\vec{E}^{A*}\cdot\vec{E}^{B})$,

$$2\sqrt{\frac{\mu}{\epsilon}}\, S_Q(t+\tau_A) = \left[S_Q(t+\tau_A) + S_Q^B(t+\tau_A)\right] 2\sqrt{\frac{\mu}{\epsilon}}$$
$$+ 2\,\mathrm{Re}\left(\vec{E}_p^{A*}(t)\cdot\vec{E}_p^{B}(t-\tau)\, e^{i\omega\tau}\right)$$

Hence

$$I_Q(t+\tau_A) = I_A + I_B$$
$$+ \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\; 2\,\mathrm{Re}\left(\vec{E}_p^{A*}(t)\cdot\vec{E}_p^{B}(t-\tau)\, e^{i\omega\tau}\right)$$

(d) Taking average for interval t part,
(i.e. long compared with t)

$$\langle I_Q^{A+B}\rangle(\tau) = I^A + I^B +$$
$$\frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\,\langle 2\,\mathrm{Re}\left(\vec{E}_p^{A*}(t)\cdot\vec{E}_p^{B}(t-\tau)\, e^{i\omega t}\right)\rangle.$$

$\boxed{3}$

(a) Just a simple substitution

(b)
$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad \left(\text{noting } -1 \le \mathrm{Re}\left(e^{i\omega t}\right) \le 1 \right)$$

$$= \frac{4 \left(I_Q^A \, I_Q^B \right)^{\frac{1}{2}} \left| \gamma^{AB}(\tau) \right|}{2 \left(I_Q^A + I_Q^B \right)}$$

If $I^A = I^B$,

$$V = \left| \gamma^{AB}(\tau) \right| .$$

(c) They are not related. Good discussion topic, though!

4

For monochromatic light,

$$\vec{E}_A = \vec{E} \exp i(\vec{k}_A \cdot \vec{r}_A - \omega t)$$

$$\vec{E}_B = \vec{E} \exp i(\vec{k}_B \cdot \vec{r}_B - \omega t), \quad \text{so}$$

$$I(x) = (\vec{E}_A + \vec{E}_B)(\vec{E}_A^* + \vec{E}_B^*)$$

$$= 2|\vec{E}|^2 (1 + \cos(\vec{k}_A \cdot \vec{r}_A - \vec{k}_B \cdot \vec{r}_B))$$

$$= 2|\vec{E}|^2 (1 + \cos kx)$$

For nonmonochromatic light, intensity $I(x)$ is integral over k for intensity at P which can be expressed as (by letting $G(k)$ to be intensity when there is no interference term (i.e. $2|\vec{E}|^2$))

$$I(x) = \int_0^\infty dk \; G(k)(1 + \cos kx)$$

$$= \int_0^\infty G(k)\,dk + \int_{-\infty}^{\infty} G(k) \frac{e^{ikx} + e^{-ikx}}{2} dk$$

Using $I(0) = \int_0^\infty dk \; G(k)(1+1)$,

$$= \frac{1}{2} I(0) + \frac{1}{2} \int_{-\infty}^{\infty} G(k) e^{ikx} \, dk$$

$\therefore$

$$\int_{-\infty}^{\infty} G(k) e^{ikx} \, dk = 2 I(x) - I(0)$$

By inverse Fourier transf^{n},

$$G(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(2 I(x) - I(0) \right) e^{-ikx} \, dx$$

$\square$

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(a) $$l_s = \frac{1.22\,\lambda}{\theta} = \frac{1.22 \times 600 \times 10^{-9}}{0.047/3600 \times \frac{\pi}{180}} = 3.2 \text{ (m)}.$$

So ν of order $\frac{1}{2}$ is expected to be around $3.2 \cdot \frac{1}{2} \sim 1.6$ (m).

(b) 21st peak of light with wave length λ coincides with 20th peak of $\lambda + \Delta\lambda$.

So $20(\lambda + \Delta\lambda) = 21\lambda$

$\therefore \Delta\lambda = 30$ nm.

$$l_c = \frac{\lambda^2}{\Delta\lambda} = 12{,}000 \text{ nm}.$$

($l_c = 20\lambda$ as we expected)

$\boxed{6}$

(a) For a quarter wave length film, (see eq^{n} (19.36))

$$R = |r|^2 = \frac{(-n_T n_0 + n_1^2)^2}{(n_T n_0 + n_1^2)^2}$$

Hence $R = 0 \longrightarrow -n_T n_0 + n_1^2 = 0$

or $n_1 = \sqrt{n_0 n_T}$

(b) So we want to compare

$$R = |r|^2 = \left| \frac{-n_T n_0 + n_1^2}{n_T n_0 + n_1^2} \right|^2$$

with the R before putting film

$$R = |r|^2 = \left| \frac{n_0 - n_T}{n_0 + n_T} \right|^2 .$$

The required condition comes from

$$\left| \frac{n_0 - n_T}{n_0 + n_T} \right|^2 > \left| \frac{-n_T n_0 + n_1^2}{n_T n_0 + n_1^2} \right|^2$$

$$\downarrow$$

$$n_0^2 < n_1^2 < n_T^2$$

$$\downarrow$$

$$n_0 < n_1 < n_T .$$

$\boxed{7}$

(a) Product of transfer matrix for 3 layers of quarter wave length is

$$\begin{bmatrix} 0 & \frac{-i}{n_1} \\ -in_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{-i}{n_2} \\ -in_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{-i}{n_3} \\ -in_3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{i n_2}{n_1 n_3} \\ \frac{n_1 n_3}{n_2} i & 0 \end{bmatrix}$$

Hence $$r = \frac{\frac{i n_2}{n_1 n_3} n_T n_0 - \frac{n_1 n_3}{n_2} i}{'' \quad + \quad ''}$$

So $r = 0 \longrightarrow \frac{n_2}{n_1 n_3} n_T n_0 - \frac{n_2 n_3}{n_2} = 0$

$$\therefore \quad \frac{n_1 n_3}{n_2} = \sqrt{n_0 n_T}$$

$\boxed{7}$

(5)

Similarly we calculate

$$\begin{bmatrix} 0 & -\frac{i}{n_1} \\ -in_1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -i/n_3 \\ -in_3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} n_3/n_1 & 0 \\ 0 & n_1/n_3 \end{bmatrix}$$

So $$r = \frac{\frac{n_3}{n_1} n_0 - \frac{n_1}{n_3} n_T}{+}$$

$$r = 0 \longrightarrow \frac{n_3}{n_1} n_0 = \frac{n_1}{n_3} n_T$$

$$\frac{n_3^2}{n_1^2} = \frac{n_T}{n_0}$$

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9

From Eq (19-53),

$$R = \left[\frac{(1.38/2.35)^{12} - 1}{(1.38/2.35)^{12} + 1} \right]^2 = 0.9933$$

or $R = 99.33\%$

(OR $T = 1 - R = 0.0067$)

b) So M now becomes

$$M = \begin{bmatrix} \left(-\frac{n_H}{n_L}\right)^3 & 0 \\ 0 & \left(-\frac{n_L}{n_H}\right)^3 \end{bmatrix} \begin{bmatrix} \left(-\frac{n_L}{n_H}\right)^3 & 0 \\ 0 & \left(-\frac{n_H}{n_L}\right)^3 \end{bmatrix}$$

$$(LH)^3 \qquad\qquad (HL)^3$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \text{ So } r = \frac{n_0 - n_T}{n_0 + n_T} = 0. \text{ or } T = 1.$$