1

In lab frame,

$$\vec{r} = (r\cos\theta\cos\varphi,\ r\cos\theta\sin\varphi,\ r\sin\theta)$$

In rest frame

$$\vec{r}' = (r\sin\theta\cos\varphi,\ r\sin\theta\sin\varphi,\ \gamma\, r\cos\theta)$$

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}'|^2}\hat{r}'$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2(\gamma^2\cos^2\theta+\sin^2\theta)}\frac{(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \gamma\cos\theta)}{\sqrt{\gamma^2\cos^2\theta+\sin^2\theta}}$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \gamma\cos\theta)}{r^2(\gamma^2\cos^2\theta+\sin^2\theta)^{3/2}}$$

Now we transform $\vec{E}'$ to $\vec{E}$ using Lorentz transf^{n}

$$\vec{E}_{\parallel} = \vec{E}'_{\parallel}$$

$$\vec{E}_{\perp} = \gamma_0(\vec{E}'_{\perp} - \beta\times c\vec{B}'_{\perp})$$

Therefore we obtain $\vec{E} = \hat{r}\,\dfrac{q}{4\pi\varepsilon_0 r^2}\dfrac{\gamma}{(\gamma^2\cos^2\theta+\sin^2\theta)^{3/2}}$ □

(b)

$$\frac{\gamma}{(\gamma^2 \cos^2\theta + \sin^2\theta)^{3/2}} = \frac{\gamma}{(\gamma^2(1-\sin^2\theta) + \sin^2\theta)^{3/2}}$$

$$= \frac{\gamma}{\gamma^3\left[(1-\sin^2\theta) + (1-(\frac{v}{c})^2)\sin^2\theta\right]^{3/2}}$$

$$= \frac{1-(\frac{v}{c})^2}{(1-(\frac{v}{c})^2\sin^2\theta)^{3/2}}$$

Hence (a) & Griffiths Eq (10.68) are same.

$\boxed{2}$

$I(t) = q_0\,\delta(t)$. So

$$\bar{A}(r,t) = \frac{\mu_0}{4\pi}\,\hat{z}\int_{-\infty}^{\infty} \frac{q_0\,\delta(t-\frac{\mathscr{r}}{c})}{\mathscr{r}}$$

(where $\mathscr{r} = \sqrt{r^2+z^2}$.)

$$= \frac{\mu_0}{4\pi}\,\hat{z}\;2\int_0^{\infty} \frac{\delta(t-\frac{\mathscr{r}}{c})}{\mathscr{r}}\,dz$$

Using $dz = \dfrac{\mathscr{r}\,d\mathscr{r}}{\sqrt{\mathscr{r}^2-r^2}}$

$$= \frac{\mu_0 q}{2\pi}\,\hat{z}\int_r^{\infty} \frac{1}{\mathscr{r}}\,\delta\left(t-\frac{\mathscr{r}}{c}\right)\frac{\mathscr{r}\,d\mathscr{r}}{\sqrt{\mathscr{r}^2-r^2}}$$

using $\delta(t-\frac{\mathscr{r}}{c}) = c\,\delta(\mathscr{r}-ct)$

$$= \frac{\mu_0 q_0}{2\pi}\,\hat{z}\,c\int_r^{\infty} \frac{\delta(\mathscr{r}-ct)}{\sqrt{\mathscr{r}^2-r^2}}\,d\mathscr{r}$$

$$\therefore \vec{A}(r,t) = \frac{\mu_0 q_0 c}{2\pi} \frac{1}{\sqrt{c^2t^2 - r^2}} \hat{z}$$

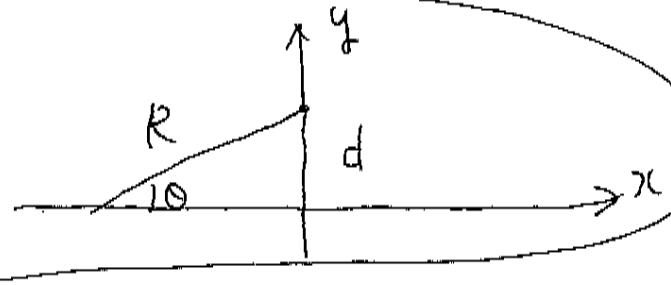
$$\vec{E}(r,t) = -\frac{\partial \vec{A}}{\partial t} = \frac{\mu_0 q_0 c^3 t}{2\pi (c^2t^2 - r^2)^{3/2}} \hat{z}$$

$$\vec{B}(r,t) = -\frac{\partial A_z}{\partial t} \hat{\phi} = \frac{-\mu_0 q_0 c r}{2\pi (c^2t^2 - r^2)^{3/2}} \hat{\phi}$$

$\boxed{3}$

(a) Eq 10.68: $\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\left(1-\frac{v^2}{c^2}\right)\int \frac{\hat{R}}{R^2}\frac{dx}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}}$

Using $d = R\sin\theta$



$$= \frac{\lambda}{4\pi\varepsilon_0}\left(1-\frac{v^2}{c^2}\right)\frac{\hat{y}}{d}\int_0^{\pi}\frac{\sin\theta}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}}\,d\theta$$

Change of variable $z = \cos\theta$

$$= \frac{\lambda\left(1-\left(\frac{v}{c}\right)^2\right)\hat{y}}{4\pi\varepsilon_0 d}\int_{-1}^{1}\frac{1}{\left(1-\left(\frac{v}{c}\right)^2+\left(\frac{v}{c}\right)^2 z^2\right)^{3/2}}\,dz$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{d}\hat{y}$$

(b) $\vec{v} = v\hat{x}$, so

$$\vec{B} = \frac{1}{c^2}\vec{v}\times\vec{E} = \frac{1}{c^2}v\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{d}(\hat{x}\times\hat{y})$$

$\lambda v = I$, therefore $\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{d}\hat{\phi}$.

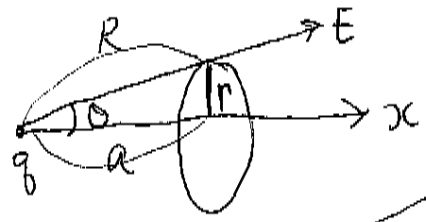
4

Poynting vector $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

Using $\vec{B} = \frac{1}{c^2}(\vec{v} \times \vec{E})$,

$$\vec{S} = \epsilon_0 \left[E^2 \vec{v} - (\vec{v} \cdot \vec{E}) \vec{E} \right]$$

Integrating over $d\vec{a} = 2\pi r \, dr \, \hat{x}$, we get power



$E_x = E\cos\theta$

$$P = \int \vec{S} \cdot d\vec{a}$$

$$= \epsilon_0 \int (E^2 v - E_x^2 v) \, 2\pi r \, dr$$

$$= 2\pi \varepsilon_0 v \int E^2 \sin^2\theta \; r \, dr$$

Eq 10.68

$$= 2\pi \varepsilon_0 v \left(\frac{q}{4\pi\varepsilon_0}\right)^2 \frac{1}{\gamma^2} \int_0^\infty \frac{r \sin^2\theta}{R^4 \left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^3} \, dr$$

$(R\cos\theta = a \;,\; a \tan\theta = r)$

$$= \frac{v}{2\gamma^4} \frac{q^2}{4\pi\varepsilon_0} \frac{1}{a^2} \int_0^{\pi/2} \frac{\sin^3\theta \cos\theta}{\left[1 - \frac{v^2}{c^2}\sin^2\theta\right]^3} \, d\theta$$

$u \equiv \sin^2\theta$

$$= \frac{v q^2}{16\pi\varepsilon_0 a^2 \gamma^4} \int_0^1 \frac{u}{\left(1 - \frac{v^2}{c^2} u\right)^3} \, du$$

$$= \frac{v q^2}{16\pi\varepsilon_0 a^2 \gamma^4} \left(\frac{\gamma^4}{2}\right) = \frac{v q^2}{32\pi\varepsilon_0 a^2}$$

$\boxed{5}$

(a) From Griffith Eq. (10.41),

$$-ct_r = \frac{+\vec{r}\cdot\vec{\beta} + \sqrt{(\vec{r}\cdot\vec{\beta})^2 - (1-(\frac{v}{c})^2)\,r^2}}{1-(\frac{v}{c})^2}$$

$$= \gamma\left(+\gamma\,\vec{r}\cdot\vec{\beta} + \sqrt{\gamma(\vec{r}\cdot\vec{\beta})^2 + r^2}\right)$$

(b) From before Eq (10.41) in Griffith,

$$R(1-\hat{R}\cdot\vec{v}/c) = \frac{1}{c}\sqrt{(\vec{r}\cdot\vec{v})^2 - (c^2-v^2)\cdot r^2}$$

$$= r\sqrt{1-\beta^2\sin^2\theta}$$

(c) $\vec{r} = \vec{R} + \vec{w}(t_r)$

$= \vec{R} + c\vec{\beta}\cdot t_r$

$= R(\hat{R} - \vec{\beta})$

$\leftarrow \vec{R} = c(\cancel{t}-t_r)$

$= -ct_r$

$\boxed{6}$

$$\frac{(\hat{R} - \vec{\beta})(1-\beta^2)}{(1 - \hat{R}\cdot\vec{\beta})^3}$$

$$= \frac{\frac{\vec{r}}{R}(1-\beta^2)}{\left(\frac{r}{R}\sqrt{1-\beta^2\sin^2\theta}\right)^3}$$

$$= \frac{\hat{r}\,(1 - v^2/c^2)}{(1 - v^2\sin^2\theta/c^2)^{3/2}}$$

at $t=0$, $\vec{R} = \vec{r}$, so

$$= \frac{\hat{R}\,(1 - v^2/c^2)}{(1 - v^2\sin^2\theta/c^2)^{3/2}}$$

Hence $\vec{E}_v = \frac{q}{4\pi\varepsilon_0}\,\frac{1 - v^2/c^2}{(1 - v^2\sin^2\theta/c^2)^{3/2}}\,\frac{\hat{R}}{R^2}$ $\square$

$\boxed{7}$

(a) $$\vec{S}_a = \frac{1}{\mu_0} \vec{E}_a \times \vec{B}_a$$

$$= \frac{1}{\mu_0} \vec{E}_a \times \frac{\left\{\hat{R} \times \vec{E}_a\right\}_{ret}}{c}$$

$$= \frac{1}{\mu_0 c} \hat{R} \left(\vec{E}_a \cdot \vec{E}_a\right) - \vec{E}_a \left(\vec{E}_a \cdot \hat{R}\right)$$

$\vec{E}_a$ is perp. to $\hat{R}$, so

$$= \frac{1}{\mu_0 c} \hat{R} \left(\frac{q}{4\pi\varepsilon_0} \frac{1}{c} \left\{ \frac{1}{R} \frac{\hat{R} \times \left[(\hat{R} - \vec{\beta}) \times \dot{\vec{\beta}}\right]}{(1 - \hat{R} \cdot \vec{\beta})^3} \right\}_{ret} \right)$$

$$= \left(\frac{q}{4\pi\varepsilon_0}\right)^2 \frac{\epsilon_0}{c} \left\{ \frac{\hat{R}}{R^2} \left[\hat{R} \times \left[\frac{(\hat{R} - \vec{\beta}) \times \dot{\vec{\beta}}}{(1 - \hat{R} \cdot \vec{\beta})^3} \right] \right]^2 \right\}_{ret}$$

(b)

$$\vec{w} = (b\cos wt,\ b\sin wt,\ 0)$$

$$c\vec{\beta} = \dot{\vec{w}} = (-bw\sin wt,\ bw\cos wt,\ 0)$$

$$c\dot{\vec{\beta}} = \ddot{\vec{w}} = (-bw^2\cos wt,\ -bw^2\sin wt,\ 0)$$
$$= -w^2\,\vec{w}$$

$$\vec{r} = (0, 0, z)$$

$$\hat{R} = \frac{1}{\sqrt{b^2+z^2}}(-b\cos wt,\ -b\sin wt,\ z)$$

$$\hat{R}\cdot\vec{\beta} = 0 \quad , \quad \hat{R}\cdot\dot{\vec{\beta}} = \frac{b^2 w^2}{c\sqrt{b^2+z^2}}$$

$$\hat{R}\times[(\hat{R}-\vec{\beta})\times\dot{\vec{\beta}}] = \hat{R}\times(\hat{R}\times\dot{\vec{\beta}}) - \hat{R}\times(\vec{\beta}\times\dot{\vec{\beta}})$$

$$\hat{R}\times(\hat{R}\times\dot{\vec{\beta}}) = \hat{R}(\hat{R}\cdot\dot{\vec{\beta}}) - \dot{\vec{\beta}}$$
$$= \frac{b^2 w^2}{\sqrt{b^2+z^2}\ c}\hat{R} - \dot{\vec{\beta}}$$

$$\hat{R} \times (\vec{\beta} \times \dot{\vec{\beta}}) = \vec{\beta}(\hat{R}\cdot\dot{\vec{\beta}}) - \dot{\vec{\beta}}(\cancel{\hat{R}\cdot\vec{\beta}})$$

$$= \frac{b^2 w^2}{\sqrt{b^2+z^2}\, c}\, \vec{\beta}$$

So

$$\frac{\hat{R}}{R^2}\left[\frac{\hat{R}\times[(\hat{R}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\cancel{\hat{R}\cdot\vec{\beta}})^3}\right]^2$$

$$= \frac{\hat{R}}{b^2+z^2}\left\{\frac{b^4 w^4}{(b^2+z^2)c^2}(-1+|\vec{\beta}|^2) + |\dot{\vec{\beta}}|^2\right\}$$

$$\sim \frac{\hat{R}}{b^2+z^2}|\dot{\vec{\beta}}|^2 \sim \frac{\hat{R}}{z^2}|\dot{\vec{\beta}}|^2$$

So $\vec{S}_a \sim \left(\frac{q}{4\pi\epsilon_0}\right)^2 \frac{\epsilon_0}{c} \frac{|\dot{\vec{\beta}}|^2}{z^2}\hat{R}$

using $|\beta| = \frac{bw}{c} \sim 1$ & $c|\dot{\beta}| = w^2 b \sim (c/b)^2 b$

$$\left(\frac{q}{4\pi\epsilon_0}\right)^2 \frac{c\,\epsilon_0}{b^2}\frac{1}{z^2}\hat{R}.$$

$\boxed{7}$ (C)

The most significant factor now is

$$\frac{1}{(1-\hat{R}\cdot\vec{\beta})^6} .$$

For $|\vec{\beta}| \sim 1$, the maximum for radiation is when $\hat{R}\cdot\vec{\beta} \sim 1$ or when $\hat{R} \parallel \vec{\beta}$.

This occurs when $\hat{R}$ is on x-y plane & $\hat{z}$ is not near the maximum direction.

$\boxed{8}$ (c is set to 1)

$$\frac{\hat{R}}{R^2} + \frac{d}{dt}\frac{\hat{R} - \vec{\beta}}{R(1-\hat{R}\cdot\vec{\beta})}$$

$$= \frac{\hat{R}}{R^2} + \frac{\left(\frac{d\hat{R}}{dt} - \frac{d\vec{\beta}}{dt}\right)\left(R(1-\hat{R}\cdot\vec{\beta})\right) - \left(\hat{R}-\vec{\beta}\right)\frac{d}{dt}\left(R(1-\hat{R}\vec{\beta}\right.}{R^2(1-\hat{R}\cdot\vec{\beta})^2}$$

The terms including $\dot{\vec{\beta}}$:

$$\frac{\dot{\vec{\beta}}\left(-R + R\hat{R}\cdot\vec{\beta}\right) + \left(\hat{R}-\vec{\beta}\right)R\,\hat{R}\cdot\dot{\vec{\beta}}}{R^2(1-\hat{R}\cdot\vec{\beta})^2}$$

$$= \frac{\dot{\vec{\beta}}\left(-1 + \hat{R}\cdot\vec{\beta}\right) + \left(\hat{R}-\vec{\beta}\right)\hat{R}\cdot\dot{\vec{\beta}}}{R(1-\hat{R}\cdot\vec{\beta})^2}$$

$$= \frac{\hat{R}\times\left[\left(\hat{R}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{R(1-\hat{R}\cdot\vec{\beta})^2}$$

The terms without $\dot{\vec{\beta}}$:

$$\frac{\hat{R}}{R^2} + \frac{\frac{d\hat{R}}{dt} R(1-\hat{R}\cdot\vec{\beta}) - (\hat{R}-\vec{\beta})\left\{\frac{dR}{dt}(1-\hat{R}\vec{\beta}) - R\frac{d\hat{R}}{dt}\vec{\beta}\right\}}{R^2(1-\hat{R}\cdot\vec{\beta})^2}$$

$$= \frac{\hat{R}}{R^2} + \frac{\frac{d\hat{R}}{dt}\left\{R(1-\hat{R}\cdot\vec{\beta}) + R(\hat{R}-\vec{\beta})\vec{\beta}\right\} + \frac{dR}{dt}(\hat{R}-\vec{\beta})(-1+\hat{R}\vec{\beta})}{R^2(1-\hat{R}\cdot\vec{\beta})^2}$$

$$= \frac{\hat{R}(1-\hat{R}\cdot\vec{\beta})^2 + \frac{\hat{R}\times(\hat{R}\times\vec{\beta})}{R}\left\{R - R\vec{\beta}^2\right\} + \hat{R}\cdot\vec{\beta}(\hat{R}-\vec{\beta})(1-\hat{R}\cdot\vec{\beta})}{R^2(1-\hat{R}\cdot\vec{\beta})^2}$$

$$= \frac{(\hat{R}\,\hat{R}\cdot\vec{\beta} - \vec{\beta})(1-\vec{\beta}^2) + (1-\hat{R}\cdot\vec{\beta})(\hat{R} - \cancel{\hat{R}\,\hat{R}\cdot\vec{\beta}} + \cancel{\hat{R}\cdot\vec{\beta}\,\hat{R}} - \hat{R}\cdot\vec{\beta}\,\vec{\beta}}{R^2(1-\hat{R}\cdot\vec{\beta})^2}$$

$$= \frac{(1-\vec{\beta}^2)(\hat{R}-\vec{\beta})}{R^2(1-\hat{R}\cdot\vec{\beta})^2}$$

Therefore

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{1-\hat{R}\cdot\vec{\beta}}\left[\frac{\hat{R}}{R^2} + \frac{d}{c\,dt}\,\frac{\hat{R}-\vec{\beta}}{R(1-\hat{R}\cdot\vec{\beta})}\right]\right\}_{ret}$$

$$= \vec{E}_a + \vec{E}_b \quad \square$$